HYDRAULIC RESISTANCE OF ROTATING TUBES

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Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 6, pp. 782–787, 1967

UDC 532.501.312

Parametric relations have been obtained, based on experimental data, for the hydraulic resistance coefficient of rotating tubes, allowing for the action of mass forces on the fluid flow.

An evaluation of the hydraulic resistance of rotating tubes is of interest in connection with the design of cooling systems for rotating machinery.

A fluid, flowing through a rotating tube, is subjected to the action of centrifugal mass forces. These forces may have an active effect on the flow, increasing the turbulence arising from external forces, or a conservative effect, leading to the suppression of turbulence and to greater stability of the flow.

An analysis of the stability of rotary motion of a fluid by the Rayleigh method shows that an isothermal stream will be stable under the condition

$$\frac{d(ur)^2}{dr} > 0, \tag{1}$$

where u is the circumferential velocity, and r is the radial coordinate.

The flow of a fluid through a rotating tube entails translational motion along the tube axis, in addition to rotation, and therefore condition (1) cannot ensure stability of the flow, but may be considered as an indication that the mass forces are conservative.

A theoretical investigation of laminar flow through a rotating tube shows [1] that solid-body rotation of the fluid is established in a stabilized stream, i.e., rotation with constant angular velocity equal to the angular velocity of the tube rotation. It may be supposed that, even in a turbulent flow, the motion is determined also by something close to the solid-body law.

When the circumferential velocities are distributed according to a solid-body law, expression (1) reduces to the condition $r > 0$, and therefore the influence of mass forces on the stream is conservative in character over the whole tube cross section.

The stabilizing effect of tube rotation on a turbulent stream has been confirmed by visual observations [2, 3]. The flow visualization was achieved by means of a dye. A turbulent flow regime was established in a tube at rest, and the flow returned to the laminar condition when the tube was rotated with a certain velocity.

The mechanism of the effect of centrifugal mass forces on turbulence in a rotating tube has not been studied, but, on the basis of an experimental investigation of turbulence in the gap between rotating cylinders [4], it may be supposed that the mass forces decrease the radial component of fluctuating velocity.

The laws for the hydraulic resistance of rotating tubes and the boundary of the fluid flow regimes have been studied experimentally. A generalization of the experimental data, the purpose of the present article, must begin by setting up a similarity shape parameter to describe the effects of mass forces on the flow. From an analysis of the differential equation of motion by the method of similarity constants, it has been established [5], that this effect may be calculated by means of the parameter

$$K = \frac{l\,\Delta F}{\rho w^2}. \tag{2}$$

The same similarity parameter is obtained from an analysis of the excess mass force formula.

The excess mass force ΔF^*, which arises during a chance radial displacement of a particle of fluid by a volume Δv with a trajectory corresponding to radius r to a trajectory with radius $r = r_0 + \Delta r (\Delta r > 0)$, is given by the formula

$$\Delta F^* = \frac{M^2}{\Delta v \rho r^3} - \frac{M_0^2}{\Delta v \rho_0 r^3}. \tag{3}$$

Following expansion of the quantity M^2/ρ in a Taylor series, and simple transformations, we obtain

$$\frac{\Delta F}{\Delta r} = \frac{1}{r^3}\left[\frac{d\rho(ur)^2}{dr} + \frac{1}{2!}\frac{d^2\rho(ur)^2}{dr^2}\Delta r + \ldots\right], \tag{4}$$

where $\Delta F = \Delta F^*/\Delta v$.

Analysis of expression (4) by the method of similarity constants also yields the parameter K (formula (2)). Thus, the equation of fluid motion and the expression for the excess mass force, which is used in analysis of flow stability, lead to the same similarity parameter.

Choosing a characteristic dimension to be the distance between points giving the maximum and minimum mass force, equal to the tube radius r, and replacing ΔF by $\rho u^2/r$ in (2), we obtain, for isothermal conditions,

$$K = \frac{u^2}{w^2} = \frac{1}{4}\left(\frac{\omega d}{w}\right)^2. \tag{5}$$

When multiplied by Re^2, parameter K for isothermal streams reduces to the parameter S:

$$S = \frac{l^3 \Delta j}{\nu^2} = \frac{1}{16}\frac{\omega^2 d^4}{\nu^2}. \tag{6}$$

The parameter S is uniquely related to the circular Reynolds number:

$$S = \frac{1}{4} Re_{circ}^2. \tag{7}$$

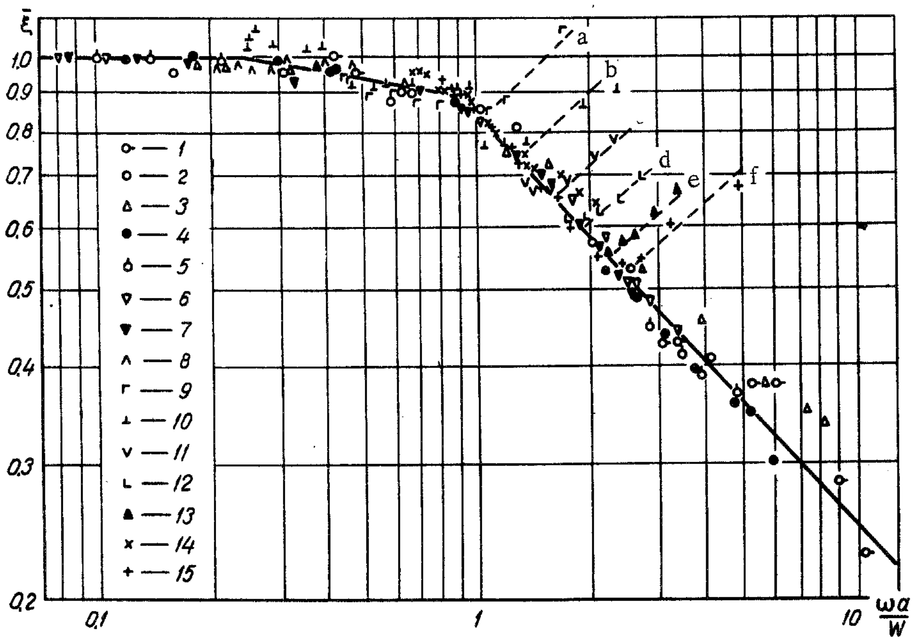

Fig. 1. Results of an experimental determination of the hydraulic resistance of rotating tubes in turbulent flow of air (1–7, according to [6]) and of water (8–15, according to [3]); 1—Re = $1.32 \cdot 10^4$, n—up to 3700 rpm; 2—$1.97 \cdot 10^4$ and up to 3200; 3—$2.44 \cdot 10^4$ and up to 4500; 4—$3.19 \cdot 10^4$ and up to 4300; 5—$4.12 \cdot 10^4$ and up to 4500; 6—$5.94 \cdot 10^4$ and up to 4500; 7—$7.05 \cdot 10^4$ and up to 4300; 8—$3.45 \cdot 10^3$—$9.47 \cdot 10^3$ and 250; 9—$2.41 \cdot 10^3$—$9.47 \cdot 10^3$ and 500; 10—$3.45 \cdot 10^3$—$3.08 \cdot 10^4$ and 1000; 11—$6.04 \cdot 10^3$—$9.47 \cdot 10^3$ and 1500; 12—$6.04 \cdot 10^3$—$9.47 \cdot 10^3$ and 2000; 13—$6.04 \cdot 10^3$—$9.47 \cdot 10^3$ and 2500; 14—10^4—$3.13 \cdot 10^4$ and 2560; 15—$5.25 \cdot 10^3$—$3.17 \cdot 10^4$ and 3120; a—n = 500 rpm; b—1000; c—1500; d—2000; e—2500; f—3120.

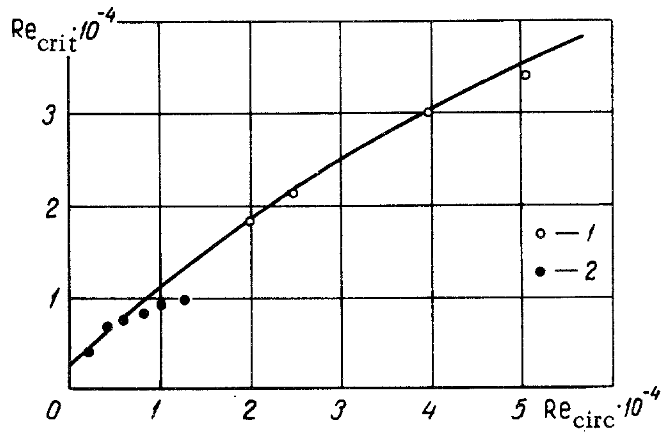

Fig. 2. Critical value of Re as a function of Re_{circ}: 1) Kas'yanov's tests; 2) White's test.

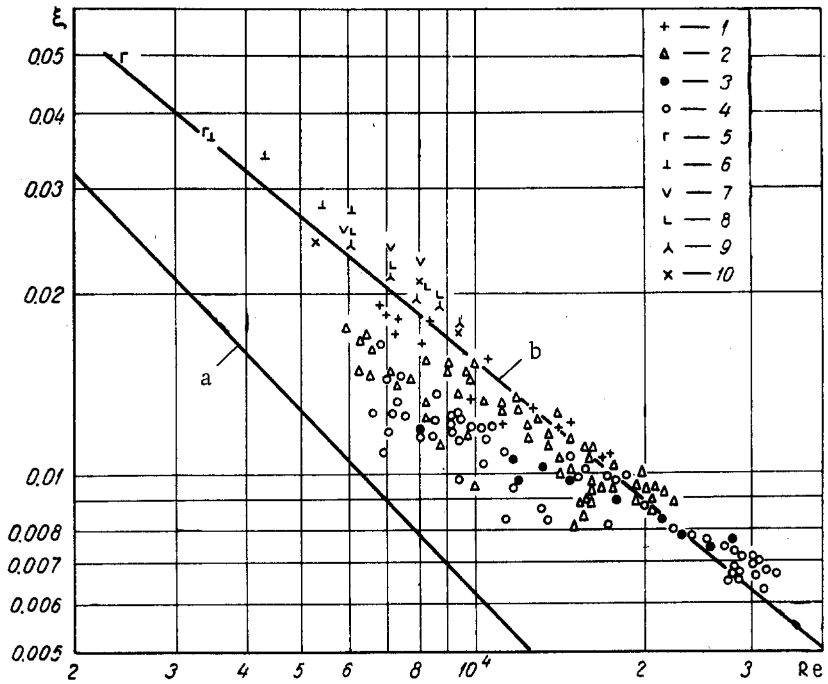

Fig. 3. The results of an experimental investigation of hydraulic resistance of rotating tubes under laminar flow of air (1–4, according to [7]) and of water (5–10, according to [3]): 1—n = 800 rpm; 2—1080; 3—1570; 4—2050; 5—500; 6—1000; 7—1500; 8—2000; 9—2500; 10—3120.

Therefore, to describe the nature of the influence of rotation on hydraulic resistance and on the boundaries of the flow regimes, we may use the parameter Re_{circ} or the parameter $\omega d/w$.

Figure 1 shows the results of investigations of hydraulic resistance of rotating tubes carried out by Levy [6] and White [3] for Re > 2300 in the form of the relation $\bar{\xi} = f(\omega d/w)$. The coefficient ξ_0, entering into $\bar{\xi}$, was determined by the Blasius law.

The experimental equipment had three sections of tube with the same diameter and a common axis: a fixed inlet tube, a rotating tube, and a fixed outlet tube. In Levy's equipment the tube had a diameter of 82 mm, and the lengths of the inlet, rotating, and outlet section were 24d, 18d, and 24d, respectively. The tests were carried out with air. White's equipment had a tube with internal diameter of 9.5 mm, the lengths of the inlet and outlet sections (from the location of pressure measurement to the rotating tube) were 28d and 100d, and the length of the rotating section was 232d. The tests were carried out in water.

In reducing the experimental data, the resistance of the fixed sections was eliminated from the total resistance by calculation, the hydraulic resistance being estimated from the Blasius formula.

It may be seen from Fig. 1 that with increase of velocity of rotation, the quantity $\bar{\xi}$ at first remains equal to unity, and then decreases. Here generalized dependence $\bar{\xi} = f(\omega d/w)$ is observed for a turbulent flow regime, independent of the value of Re. The sharp deviation from this relation (the dashed lines), which occurs at n = idem and for an increase in the parameter $\omega d/w$, may be considered as transition to a laminar flow regime.

The critical values of the parameter Re, determined from the bend in the relation $\bar{\xi} = f(\omega d/w)$, are compared (Fig. 2) with the results of an investigation of critical regimes in a rotating tube carried out by Kas'yanov [7]. Kas'yanov's tests* were carried out in air. The relation $\xi = f(\mathrm{Re})$ for the laminar regime proved to be general for all rotational velocities, and the onset of turbulence was characterized by a sharp increase in hydraulic resistance in comparison with the laminar flow case.

It may be seen from Fig. 2 that the dependence of Re_{crit} on Re_{circ}, as found by means of Fig. 1 and as determined in Kas'yanov's tests, is in satisfactory agreement and is described by the general formula

$$Re_{crit} = 7.16\, Re_{circ}^{0.78} + 2300. \tag{8}$$

This formula corresponds to the solid line in Fig. 2.

Thus, Figs. 1 and 2 show that tube rotation delays the onset of turbulence and in a turbulent stream it suppresses the development of fluctuating motion and therefore substantially decreases the hydraulic resistance of the flow.

Analysis of Fig. 1 allows us to make quantitative conclusions regarding the hydraulic resistance of rotating tubes for the turbulent flow regime. For $\omega d/w \le 0.25$ rotation has no influence on the hydraulic resistance of the tube ($\xi = 1$), and therefore the hydraulic resistance coefficient ξ is determined by the Blasius law.

For $\omega d/w = 0.25$–0.95

$$\bar{\xi} = 0.890 \left(\frac{w}{\omega d}\right)^{0.086} \quad \text{or} \quad \xi = \frac{0.281}{\mathrm{Re}^{0.25}} \left(\frac{w}{\omega d}\right)^{0.086}. \tag{9}$$

For $\omega d/w = 0.95$–10

$$\bar{\xi} = 0.865 \left(\frac{w}{\omega d}\right)^{0.535} \quad \text{or} \quad \xi = \frac{0.273}{\mathrm{Re}^{0.25}} \left(\frac{w}{\omega d}\right)^{0.535}. \tag{10}$$

Formulas (9) and (10) correspond to the solid lines on Fig. 1.

Figure 3 shows the relationship $\xi = f(\mathrm{Re})$, constructed from the results of investigation of hydraulic resistance of rotating tubes in laminar flow [7], as well as White's test data, which are joined by dashed lines on Fig. 1. The line "a" corresponds to the Poiseuille formula for fixed tubes.

Figure 3 shows that the test data exhibit considerable scatter, but no separation of the data points according to rate of rotation is observed. The figure shows that the hydraulic resistance of a rotating tube under laminar conditions is larger than that of a fixed tube.

The causes of the scatter of the test data were not given, but it may be assumed that one cause is vibration of the rotating tube due to lack of coincidence between the axis of the fixed and rotating sections.

The available test data on hydraulic resistance of rotating tubes in laminar flow are insufficient for a dependence to be obtained, and this question therefore requires additional study. However, for an estimate of the upper limit of the possible value of the hydraulic resistance coefficient, for $\mathrm{Re} = 2.3 \cdot 10^3$–$3.5 \cdot 10^4$, we may recommend the formula

$$\xi = \frac{24.1}{\mathrm{Re}^{0.8}}, \tag{11}$$

which corresponds to the line "b" on Fig. 3.

NOTATION

d is the tube internal diameter; F is the mass force referred to unit volume; F* is the mass force applied to a fluid particle; ΔF is the excess mass force (difference between the mass forces of two characteristic points of the system); K is the similarity parameter; Δj is the difference between the inertia accelerations at two points of the system; l is the characteristic dimension; M is the moment of momentum of a fluid particle; n is the number of revolutions of the tube in unit time; r is the radial coordinate, tube radius; Re is the Reynolds number, based on the mean mass velocity of the fluid motion; Re_{circ} is the Reynolds number based on the circular velocity of fluid

*Reference [7] gives no information on the dimension of the experimental tubes, but the graphs of $\xi = f(\mathrm{Re})$ obtained in the experimental equipment at rest in laminar and turbulent regimes indicate that the tube was long enough and that there was no influence of the initial section on the hydraulic resistance.

motion; S is the similarity parameter; u is the circular velocity of the inner surface of the tube; Δv is the volume of a fluid particle; w is the mean mass velocity of the fluid; ν is the kinematic viscosity; $\bar{\xi} = \xi/\xi_0$; ξ is the coefficient of hydraulic resistance of a rotating tube; ξ_0 is the same for a fixed tube; ρ is the fluid density; ω is the angular velocity of rotation of the tube.

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28 September 1966 Kazan Aviation Institute